Homework 8

Learning goals: (1) Numerically implement the discrete map for given parameter values; (2) Generate a numerical bifurcation diagram; (3) Explore the period-doubling route to Chaos

Consider the logistic map

$$x_{n+1} = \mu x_n(1 - x_n) \tag{1}$$

where $0 < x_0 < 1$ and $\mu > 0$ is a parameter. Recall from class this map exhibits bifurcations at $(\mu^*, x^*) = (1, 0)$ and $(3, \frac{2}{3})$.

(1) **Iterate the logistic map** In your script, choose an initial condition $x_0 \in (0, 1)$, e.g., $x_0 = 0.1$.

(a) For $\mu = 0.5$, write a script to iterate the logistic map. Hint: set `x(1)=x0` and use a `for` loop to update $x(n+1)$ for $n = 1, \ldots, 1000$. Store the values in a vector `x`.

(b) Plot the sequence $x(n)$ versus n (time series plot). Label the axes clearly using `xlabel` and `ylabel`.

(c) Repeat for $\mu = 2$, $\mu = 3.2$, $\mu = 3.5$, $\mu = 3.8$ and $\mu = 4$. Use `subplot` to display all six time series in a single figure. If details are hard to see, use `xlim` to zoom.

(d) In a separate figure, plot the *Poincaré map* by scattering $(x(n), x(n+1))$ for each μ (again using six subplots).

Your submission for part (1): Attach the figure with six time series subplots and the figure with six poincare subplots. Briefly describe what you observe. For each value of μ, does the orbit converge to a fixed point, a 2-cycle, a 4-cycle, or something more complicated (e.g., chaos)?

(2) **Generate numerical bifurcation diagram and explore the period-doubling route to Chaos**

In this part, you will compute an approximate bifurcation diagram of the logistic map by sweeping over values of μ.

(a) Choose a range of μ values $\mu \in [2, 4]$. Create a vector `muvals` of N equally spaced points in this interval (for example, `N = 100`):

```
mu_min = 2; mu_max = 4;
N = 100;
muvals = linspace(mu_min, mu_max, N);
```

(b) For each μ in `muvals`, iterate the logistic map starting from the same initial condition, e.g., $x_0 = 0.1$, for a total of 1000 steps. To approximate the long-term behavior, discard the first 500 iterates (transient) and keep the last 500 iterates.

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(c) Scatter the *kept* iterates (μ, x_n) as points in the (μ, x)-plane.

Your submission for part (2):

- Attach a screenshot of your MATLAB code for generating the bifurcation diagram. Be sure to include comments explaining the key steps.
- Include the resulting bifurcation diagram. Label the axes clearly.
- Describe the structures you observe as μ increases (e.g. μ intervals where fixed points exist, the onset of period-doubling, chaotic regimes, windows of periodic behavior, etc.). Compare these features with your plots from Part (1) and comment on whether your observations are consistent with the bifurcation diagram covered in class.