Homework 6

Key concepts: discrete difference equations, analysis of fixed point and periodic orbits in discrete dynamical systems, simple bifurcations

This HW will not be collected or graded. It's intended as a practice problem set on the key concepts listed above. For review of earlier materials, refer to previous HWs and lecture notes.

1. Consider the following logistic population model

$$x_{n+1} = F(x_n)$$

   where the function $F(x) = rx(1-x)$.

   (2) Let $r = 2$. Find all the fixed points and identify the stable set for each fixed point.

   (3) Show that a bifurcation occurs at $r = 1$.

   (4) Draw a graph of the function F, the identity line and sample orbits for r smaller than, equal to, and larger than the bifurcation point $r = 1$. Describe how the solutions change at the bifurcation.

2. Consider the equation $x_{n+1} = f(x_n)$ where $f(x) = -x^3$

   (a) Find the fixed points and the corresponding stable sets

   (b) Find periodic points with period 2. Use the graph of f and the identify line to sketch the period-2 periodic orbit. Determine their stability.

3. Find all periodic points for the map $f(x) = x^3 - x$ and classify them as attracting, repelling, or neither. Sketch the phase portraits.

4. Consider the map $f_c(x) = x^2 + c$. We discussed in class that a saddle-node bifurcation occurs at $c = \frac{1}{4}$. Can you find the other bifurcation, and describe how the solutions change at that bifurcation.