

MATH 37a Differential Equations **Homework 8**

This homework covers sections 6.2, 6.3 and 6.4

1. Find the inverse Laplace transform of $F(s) = \frac{2s-3}{s^2-4}$.
2. Find the inverse Laplace transform of $F(s) = \frac{8s^2-4s+12}{s(s^2+4)}$.
3. Use the Laplace transform to solve

$$y^{(4)} - 4y = 0, \quad y(0) = 1, y'(0) = 0, y''(0) = -2, y'''(0) = 0.$$

4. Use the Laplace transform to solve

$$y'' - 2y' + 2y = \cos t, \quad y(0) = 1, y'(0) = 0.$$

5. Sketch the graph of $g(t) = (t-1)u_1(t) - 2(t-2)u_2(t) + (t-3)u_3(t)$ on $t \geq 0$
6. Consider

$$f(t) = \begin{cases} 0, & 0 \leq t < 3 \\ -2, & 3 \leq t < 5 \\ 2, & 5 \leq t < 7 \\ 1 & t \geq 7 \end{cases}$$

- (a) Sketch its graph
 - (b) Express $f(t)$ in terms of the unit step function $u_c(t)$.
7. Find the Laplace transform of $f(t) = (t-3)u_2(t) - (t-2)u_3(t)$.
 8. Find the inverse Laplace transform of $F(s) = \frac{2(s-1)e^{-2s}}{s^2-2s+2}$
 9. Find the solution of $y'' + 3y' + 2y = f(t), y(0) = y'(0) = 0$ where

$$f(t) = \begin{cases} 1, & 0 \leq t < 10 \\ 0 & t \geq 10 \end{cases}$$

10. Find the solution of $y'' + y = g(t), y(0) = 0, y'(0) = 1$ where

$$g(t) = \begin{cases} t/2, & 0 \leq t < 6 \\ 3 & t \geq 6 \end{cases}$$